If the input and output VSWR of the component are equal and the error is small, the limit of mismatch error may be written as

$$\lim \epsilon_e = 2\,|S_{11}| \text{ radians} \tag{33}$$

which can be readily shown to be equivalent to a result for lossless components quoted by Magid.[2] It may be useful here to emphasize the meaning of the limits of error obtained by (33). These are maximum errors due to mismatches, since it was assumed that the phase changes of all coefficients were arbitrary.

[2] M. Magid, "Precision microwave phase shift measurements," IRE Trans. on Instrumentation, vol. I-7, pp. 321–331; December, 1958.

For the same phase shifter considered in Application I (VSWR<1.35), the limits of error are ±17°. It is of interest to note that the mismatch error in a variation of phase shift measurement in this method is independent of the reflection coefficient of the generator.

Additional errors in this method such as those caused by probe loading in the slotted line are not within the scope of this analysis, but should be taken into account, if they are appreciable compared to the mismatch error.

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A Note on the Optimum Source Conductance of Crystal Mixers*

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Summary—**This paper describes an accurate and convenient technique for measuring the match of a crystal mixer. Use is made of the fact that with a proper RF drive level, the fundamental conductance of a mixer crystal may be made equal to the optimum source conductance of the crystal for mixer operation. The required drive level depends on certain crystal parameters and on the image frequency termination of the mixer. Design curves are given which simplify the determination of the proper RF drive level for a wide range of crystal parameters and their condition of image frequency termination.**

INTRODUCTION

THE DESIGN of a crystal mixer may conveniently be broken down into three parts:

1) design of a signal coupling mechanism which will provide the optimum source conductance for minimum available conversion loss,
2) design of a local-oscillator coupling mechanism that has negligible effect on the signal admittance,
3) design of an RF bypass circuit that will not allow the RF power to couple to the IF load circuit.

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This paper deals with certain considerations of the design of the signal coupling mechanism. In order to achieve a minimum noise figure crystal mixer receiver, it is necessary to design the mixer for minimum available conversion loss. From linear network theory, applicable to a crystal mixer, there is an optimum source conductance for minimum available conversion loss. In the practical design of a crystal mixer, it is usually assumed that this optimum conductance is equal to the fundamental component of the conductance of the crystal for a high-level RF signal, of the same magnitude as the local-oscillator drive, but in the absence of this local-oscillator drive. Under this assumption, the crystal mount is then designed to be matched to the line at this high level of RF signal.[1] This generally gives a good approximation to the optimum match condition for the broadband mixer[2,3] (image frequency termination equal

[1] R. V. Pound, "Microwave Mixers," Rad. Lab. Series, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 16, p. 122; 1945.
[2] H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers," Rad. Lab Series, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 15, pp. 111–178; 1948.
[3] Operation in this condition is discussed by Torrey and Whitmer, *Ibid.* Data cited there show that the conversion loss (L_0) for this condition differs from the optimum conversion loss (L_2) in the broadband condition by less than 0.2 db.

to the signal frequency termination). However, the question arises as to how accurate this technique is in approximating the optimum match condition for narrow-band mixers (image frequency termination short-circuited or open-circuited).

It is the purpose of this paper to investigate the accuracy of this measurement method for the three mixer cases of general interest:

1) short-circuited image frequency termination,
2) image frequency termination = signal frequency termination,
3) open-circuited image frequency termination;

and to show how, from a knowledge of the dc characteristics of the crystal, a modification in the RF drive level can be made to improve the accuracy. Only single-ended mixers will be discussed. However, application to balanced mixers should be clear.

The interrelationship of the high-level crystal conductance and the optimum mixer source conductance may not be immediately apparent. For this reason, mixer theory will be briefly reviewed and equivalent networks will be presented which will show this relationship quite clearly.

Review of Crystal Mixer Theory

A simple mixer circuit is shown in Fig. 1. The RF choke serves as a low-impedance path for both dc and the IF frequency. Hence, as viewed from the IF terminals, the crystal is effectively across BB'. The RF bypass assures that no signal power will be lost to the IF circuit and, as viewed from the RF terminals, effectively places the crystal across AA'. The local oscillator is represented by the current source $i_0 = I_0 \cos \omega_0 t$, and admittance Y_0. In a practical mixer circuit, the local oscillator is very loosely coupled to the crystal ($Y_0 \ll Y_r$, where Y_r = signal generator admittance), and its effect on the conversion loss and impedance relationships can be neglected. The area indicated as "crystal" is meant to include only the active region of the crystal, that is, the vicinity of the point contact. All the passive parts of the crystal (whisker, ceramic and contacts) are implicit in the RF transformation network. Consequently, all admittances shown are referred to this region.

It has been shown[2,4,5] that a crystal mixer can be accurately represented, at the active region of the crystal, by the three-port network shown in Fig. 2. The conductances of the network can be obtained from an analysis of the current that will flow as a result of a small signal voltage being applied to the crystal, when the crystal is excited by a high-level local oscillator signal. Except for the fact that frequency translations implicitly occur, the laws of linear network theory apply to the network. An analysis of the network will yield, among other parameters, the available conversion loss, and optimum source conductance for minimum available conversion loss, for any arbitrary image frequency termination. Any deviation of the source conductance from its optimum value will cause a degradation in conversion loss by approximately the mismatch loss.

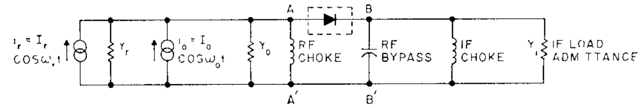

Fig. 1—Equivalent circuit of simple mixer.

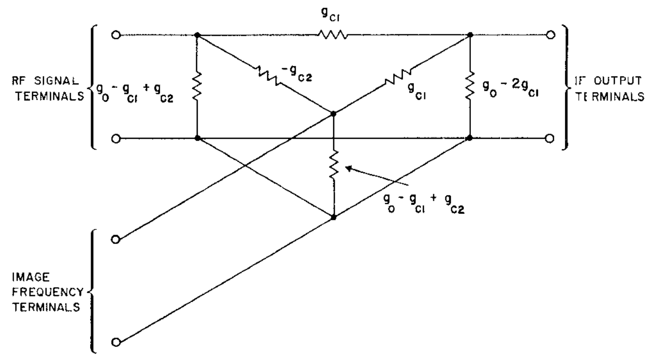

Fig. 2—Equivalent network of crystal for mixer calculations.

From Fig. 2, it can be seen that the image frequency termination will affect the optimum source conductance and conversion loss. The conditions of image termination of general interest are:

Case 1) image frequency terminals shorted (narrow-band case),
Case 2) image frequency termination equal to signal frequency termination (broadband case),
Case 3) image frequency terminals open-circuited (narrow-band case).

For mixer crystals with small back conductance, it has been shown[6] that the open-circuited image condition (Case 3) will always yield the greatest conversion efficiency. It should be noted that, in a properly designed mixer, the over-all improvement in receiver performance obtained by using an open-circuited image frequency termination in preference to a matched image frequency termination is not large, usually about 0.5 to 1.0 db decrease in over-all noise figure. However, in cases where a preselector must be used for purposes other than noise figure considerations, improper location of the preselector relative to the mixer can increase

[4] L. C. Peterson and F. B. Llewellyn, "The performance of mixers in terms of linear network theory," Proc. IRE, vol. 33, pp. 458–476; July, 1945.

[5] E. W. Herold, R. R. Bush, and W. R. Ferris, "Conversion loss of diode mixers having image frequency impedance," Proc. IRE, vol. 33, pp. 603–609; September, 1945.

[6] P. D. Strum, "Some aspects of mixer crystal performance," Proc. IRE, vol. 41, pp. 875–889; July, 1953.

over-all receiver noise figure by as much as 4.0 to 5.0 db over that which would be obtained if the preselector were properly placed. This degradation is primarily due to:

1) increased conversion loss,
2) increased IF noise figure resulting from the effect of a radically changed IF output impedance.

The optimum source conductance for each case of image frequency termination may be expressed in terms of the conductances of the network of Fig. 2. These are:

Case 1

$$g_{r1} = \sqrt{g_0^2 - g_{c1}^2} \tag{1}$$

(Image short-circuited),

Case 2

$$g_{r2} = \sqrt{(g_0 + g_{c2})\left(g_0 + g_{c2} - \frac{2g_{c1}^2}{g_0}\right)} \tag{2}$$

(Image matched),

Case 3

$$g_{r3} = \sqrt{g_0^2 - g_{c2}^2}\sqrt{\frac{2g_{c1}^2(g_{c2} - g_0) - g_0(g_{c2}^2 - g_0^2)}{g_0(g_0^2 - g_{c1}^2)}} \tag{3}$$

(Image open-circuited),

where

$$g_0 = \frac{1}{2\pi}\int_0^{2\pi} g\,d\omega_0 t, \tag{4}$$

$$g_{cn} = \frac{1}{2\pi}\int_0^{2\pi} g \cos 2n\omega_0 t\,d\omega_0 t \tag{5}$$

and g = instantaneous small-signal conductance.

These conductances, g_0, g_{c1} and g_{c2}, are found from an analysis of the current flow resulting from the application of a low-level signal to a crystal driven by the local oscillator, and, as discussed in the next section, may conveniently be expressed in terms of the parameters of the crystal law.

Important Mixer Parameters in Terms of the Crystal Law

The E–I characteristics of a crystal can be very closely approximated by:[6]

$$i_f = Ke^x; \qquad (e > 0) \tag{6}$$

$$i_b = g_b e; \qquad (e < 0), \tag{7}$$

where i_f is instantaneous forward current, i_b is the instantaneous back current, e is the applied voltage, k is a proportionality constant, x is the approximate value of $d \log i_f / d \log e$ of the crystal, and g_b is the nearly constant back conductance. (The method of obtaining these parameters from the dc characteristic of a crystal is illustrated in the Appendix.)

The instantaneous small-signal conductance is:

$$g = \frac{di}{de} = Kxe^{x-1}; \qquad (e > 0) \tag{8}$$

$$g = g_b; \qquad (e < 0). \tag{9}$$

Assuming a local-oscillator voltage of the form $e = E_0 \cos \omega_0 t$, and substituting g from (8) and (9) into (4) and (5), we get

$$g_0 = A\left[\frac{\left(\frac{x-2}{2}\right)!}{\left(\frac{x-1}{2}\right)!}\right] + \frac{g_b}{2} \tag{10}$$

$$g_{c1} = A\left[\frac{\left(\frac{x-1}{2}\right)!}{\left(\frac{x}{2}\right)!}\right] + \frac{g_b}{\pi} \tag{11}$$

$$g_{c2} = A\left[\frac{2\left(\frac{x}{2}\right)!}{\left(\frac{x+1}{2}\right)!} - \frac{\left(\frac{x-2}{2}\right)!}{\left(\frac{x-1}{2}\right)!}\right] \tag{12}$$

where

$$A = \frac{KxE_0^{x-1}}{2\sqrt{\pi}}.$$

Other quantities of interest are the fundamental component of the high RF level conductance, g_1 (which is the ratio of the fundamental component of the crystal current to the applied voltage, and is the conductance which is used to approximate the optimum source conductance required for minimum L_c) and the rectified current I_a[7]:

$$g_1 = g_0 - g_{c2} \tag{13}$$

$$I_a = \frac{KE_0^x}{2\sqrt{\pi}}\frac{\left(\frac{x-1}{2}\right)!}{\left(\frac{x}{2}\right)!} - \frac{g_b E_0}{\pi}. \tag{14}$$

Application to Matching Technique

Based on Figs. 1 and 2, the equivalent circuit of Fig. 3 is obtained. As in Fig. 1, the passive parts of the crystal structure are implicit in the various transformers

[7] S. Okwit, "Theoretical Analysis of High-Q Mixer Receiver Design," M.S. thesis, Adelphi College, Garden City, N. Y.; 1957

shown. Physically, the transformers for the signal and image frequencies are identical. However, in general, the RF matching network is frequency-sensitive and the representation of Fig. 3 shows this explicitly. The characteristic admittance, g_r, of the input transmission line is assumed to be the same at the signal frequency, ω_s, and at the image frequency, $\omega_k = 2\omega_0 - \omega_s$.

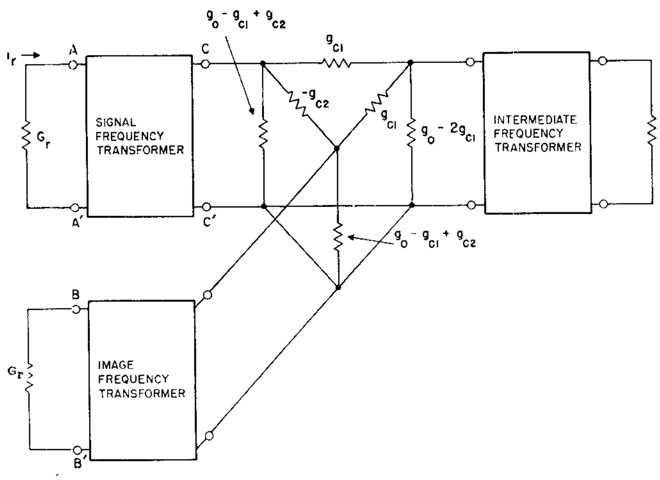

Fig. 3—Equivalent circuit of mixer.

For minimum conversion loss, the conductance seen looking toward the generator at terminals CC' should be equal to the optimum source conductance for the mixer [(1)–(3)]. A convenient way to obtain this optimum match condition in practice is first to place a conductance at the active region of the crystal (terminal CC', Fig. 3) which is equal to the optimum source conductance and then to adjust the input transformation network to obtain optimum power transfer to the conductance. This corresponds to a conjugate match at terminal CC', and if the signal frequency transformer is lossless then there will also be a condition of match at terminals AA'. Hence, slotted-line measurements in the input transmission line can accurately record the degree of match of the crystal mixer. These data can then be used to "broadband" match the crystal mount by the use of conventional microwave broadband matching techniques.[8]

A convenient method of effectively placing the optimum conductance at the active region of the crystal is to drive the crystal with a predetermined high level of RF, such that its fundamental conductance g_1 (13), is equal to the optimum source conductance of the mixer for the particular case of interest [(1)–(3)]. The level of the drive is indicated in practice by the amount of rectified current. In the VSWR measurement, it is only necessary that the probe in the slotted section be sufficiently narrow-band that the second- and higher-order harmonics of the rectified current be rejected adequately.

[8] G. L. Ragan, "Microwave Transmission Circuits," Rad. Lab. Series, McGraw-Hill Book Co., Inc., vol. 9; 1948.

Let us now discuss how the foregoing is applied to the design of a crystal mixer. Consider the following. It is desired to design a broad-band mixer (Case 2) using a 1N21E crystal. The drive level to be used for optimum sensitivity, as recommended by the manufacturer, is 0.6 ma. It is necessary for the designer to make measurements of the E–I characteristics of several crystal samples, in order to obtain the average crystal law parameters. Utilizing the techniques described in the Appendix, we find the average crystal law parameters of the 1N21E crystal to be:

$$X = 4$$
$$K = 0.4$$
$$g_b = 0.2 \text{ millimhos.}$$

It now remains to determine at what level of RF drive (rectified current) the crystal mount should be matched to the line.

The procedure is as follows. From the parameters of the crystal law the optimum source conductance, g_{r2}, is determined. A value of rectified current is then found at which g_1 is equal to g_{r2}. This will then be the level of RF drive at which the crystal mount should be matched to the line.

From (10)–(13),

$$g_0 = 0.340E_0^3 + 10^{-4} \tag{15}$$
$$g_{c1} = 0.300E_0^3 - 0.637 \times 10^{-4} \tag{16}$$
$$g_{c2} = 0.204E_0^3 \tag{17}$$
$$g_1 = 0.136E_0^3 + 10^{-4}. \tag{18}$$

The value of E_0, as determined from (14), with a crystal drive of 0.6 ma is $E_0 = 0.307$ volts. Use of this value of E_0 in (15)–(17) yields

$$g_0 = 9.92 \text{ millimhos} \tag{19}$$
$$g_{c1} = 8.62 \text{ millimhos} \tag{20}$$
$$g_{c2} = 5.90 \text{ millimhos.} \tag{21}$$

Substituting (19)–(21) into (2), we get

$$g_{r2} = 3.68 \text{ millimhos.} \tag{22}$$

The level of rectified current needed to make the fundamental conductance of the crystal equal to the optimum source conductance (22) can now be found. Eq. (18) is solved yielding $E_0 = 0.298$ volts, at which value $g_1 = g_{r2} = 3.68$ millimhos.

From (14), $I_a = 0.53$ ma, which is the required value of crystal drive for correct matching as compared to the usual 0.6-ma drive which would be used by the standard method. If on-off square-wave modulated RF is used in the measurement, the average current, as read on a milliammeter, should be half this value.

A similar procedure applied to the short-circuited and open-circuited image frequency conditions yields a crystal drive of 0.81 ma and 0.42 ma, respectively, for correct matching.

It is of interest to see what mismatches would exist if the mixer were matched to the line at a signal level equal to the operating local-oscillator level. For the 1N21E crystal utilized in the previous example, (18) yields $g_1 = 4$ millimhos at 0.6 ma drive. In the broadband case, the mixer would have been mismatched by $4/3.68 = 1.09:1$; and in the narrow-band conditions $4.92/4 = 1.23:1$ for Case 1 and $4/3.01 = 1.33:1$ for Case 3. These mismatches are, in themselves, small and produce little degradation in conversion loss. However, when this mismatch is considered together with the usual crystal impedance spread and accuracy of adjustment of match over a sizeable band, it is recognized that the combination VSWR may be significantly greater than that which would be obtained by matching with the correct drive level.

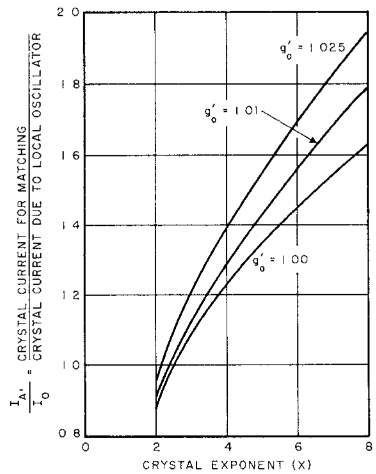

Fig. 4—Normalized crystal drive current vs crystal exponent for Case 1 (short-circuit-image).

General Design Curves

By making certain simplifying approximations, we may obtain a set of design curves which will greatly simplify the procedure described above.

For $g_b \ll g_0$, as in most high-performance crystals, it can be shown that

$$\frac{I_{on}}{I_{a0}} \simeq \left(\frac{g_{rn}}{g_1}\right)^{x/x-1}, \tag{23}$$

where

I_{an} = rectified crystal current at which fundamental conductance $= g_{rn} (n = 1, 2, 3)$,

I_{a0} = rectified crystal current at which crystal will be operated in practice,

g_{rn} = optimum source conductance for case n $(n = 1, 2, 3)$,

g_1 = fundamental conductance at drive level I_{a0}.

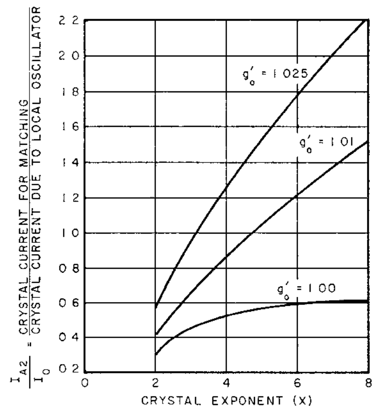

Fig. 5—Normalized crystal drive current vs crystal exponent for Case 2 (matched image).

By use of the above equations and the calculated values of g_{rn} for a wide range of x and g_0', the curves of Figs. 4–6 were obtained. These curves cover most of the high-performance crystals available today.

The curves are plotted with g_0' as a parameter, which is the average small-signal conductance normalized to the average small-signal conductance in the forward direction. g_0' is [from (10)]

$$g_0' = \frac{g_0}{g_0 - g_b/2} \simeq 1 + \frac{g_b}{2g_0} \cdot \tag{24}$$

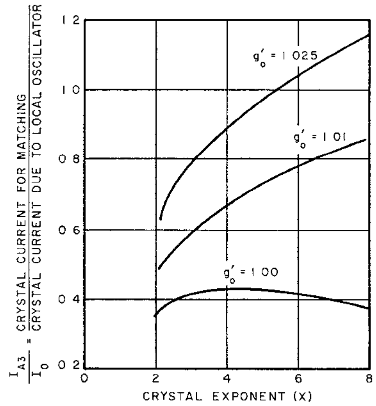

Fig. 6—Normalized crystal drive current vs crystal exponent for Case 3 (open-circuited image).

Applying these curves to the previous example, we get the following approximate values:

Case 1 $I_{a1} = 1.3 \times 0.6 = 0.780$ ma
Case 2 $I_{a2} = 0.88 \times 0.6 = 0.528$ ma
Case 3 $I_{a3} = 0.67 \times 0.6 = 0.402$ ma,

which compare favorably with the exact values in the example.

Conclusions

From a knowledge of the dc characteristics of a mixer crystal, a drive level may be found at which the crystal impedance is equal to the optimum source impedance of the mixer. The optimum drive level has been calculated for a wide range of crystal parameters for the three important cases of image frequency termination. The resultant design data are presented in Figs. 4–6.

Although very little experimental verification is available, the design data presented here are based on generally accepted mixer theory, that is, the validity of the equivalent network of the mixer and the validity of the relation of the various crystal conductances to the dc characteristics of the crystal.

Appendix

Determination of Parameters of Crystal Law

Inspection of the E–I characteristics of a mixer crystal plotted on logarithmic coordinates reveals that the crystal law may be approximated over a large portion of the current range by

$$i_f = ke^x \qquad (e > 0) \tag{25}$$

$$i_b = g_b e \qquad (e < 0), \tag{26}$$

where

i_f = instantaneous forward current
i_b = instantaneous back current
e = instantaneous voltage
x = nearly constant slope = $(d \log i)/d \log e$
g_b = nearly constant back conductance
k = proportionality constant.

Fig. 7, shows the E–I crystal characteristics of a typical 1N21E crystal plotted on logarithmic coordinates. From Fig. 7, it is seen that over any 10 to 1 range in current (in the region between 0.05 ma and 5 ma) the E–I curve can be approximated by a straight line. This, fortunately, is the region of most interest in crystal mixer operation.

Determination of x

The average crystal slope is

$$x = \frac{d \log i}{d \log e} = \frac{\log \dfrac{i_1}{i_2}}{\log \dfrac{e_1}{e_2}}$$

where $i_1 > i_2$, $e_1 > e_2$.

By use of a 10 to 1 current range,

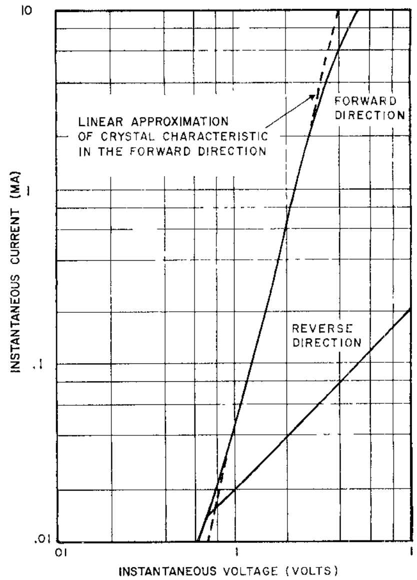

Fig. 7—E-I characteristics of 1N21E crystal.

$$x = \frac{1}{\log \dfrac{e_1}{e_2}}. \tag{27}$$

Application of (27) to the 1N21E crystal plotted in Fig. 7 (where the 10 to 1 current range is 0.1 to 1.0 ma) yields:

$$X = 4.$$

Determination of K

Setting $e = 1$ volt in (25) yields $i_f = K$. Hence, by extending the straight-line approximation of the curve to $e = 1$ volt, K is determined graphically as the current at the intercept of the 1 volt axis. Application of the above to Fig. 7 yields:

$$K = 0.4.$$

Determination of g_b

Setting $e = 1$ volt in (26) yields $i_b = g_b$. For the crystal characteristics plotted in Fig. 7,

$$g_b = 0.2 \text{ millimhos.}$$

In actual operation, the equivalent parameters of the crystal law may be modified by 1) the effect of dc impedance in the crystal circuit which will cause a bias, and 2) the effect of RF impedance in series with the crystal at harmonics of the local-oscillator frequency which will cause the applied local-oscillator voltage to be nonsinusoidal. These effects are usually small and in a direction to cancel each other. They have been neglected in this paper.